RESEARCH ARTICLE

Onintuitionistic fuzzy-*I*-open sets, intuitionistic fuzzy semi-*I*-open sets and a decomposition of intuitionistic fuzzy-*I*-continuity

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ABSTRACT: In this paper, we introduce a new class of intuitionistic fuzzy ideal open sets in intuitionistic fuzzy ideal topological spaces; intuitionistic fuzzy-*I*-open set and intuitionistic fuzzy semi-*I*-open set and given a decomposition of fuzzy continuity.

KEYWORDS: Intuitionistic fuzzy-*I*-open set, intuitionistic fuzzy semi-*I*-open set, intuitionistic fuzzy-*I*-continuous, intuitionistic fuzzy semi-*I*-continuous.

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INTRODUCTION

The concept of fuzzy sets is an important concept in many fields. In 1965, the concept was introduced [1]. Accordingly, Chang [2] introduced the concept of fuzzy topological spaces. The generalizations notion of fuzzy set has been studied and reported in many studies. Atanassov [3-5] reported the ideal of intuitionistic fuzzy set (IFS, in short). Subsequently, Coker and Saadati [6, 7] defined the notion of intuitionistic fuzzy topology and studied the basic concept of intuitionistic fuzzy point. The present article aims to extend those ideal of general topology inintuitionistic fuzzy topological spaces(IFTS, in short). Salama and Alblowi [8] introduced the notions of intuitionistic fuzzy ideal and intuitionistic fuzzy local function in intuitionistic fuzzy set theory.The intuitionistic fuzzy sets useful in medical diagnosis using three steps such as; determination of symptoms, formulation of medical knowledge based on intuitionistic fuzzy relations, and determination of diagnosis based on composition of intuitionistic fuzzy relations. Intuitionistic fuzzy set is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc. since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. Many applications of intuitionistic fuzzy set is carried out using distance measures approach.

In this paper, we define intuitionistic fuzzy-*I*-open set and intuitionistic fuzzy semi-*I*-open set via intuitionistic fuzzy ideal topology. Moreover, we define intuitionistic fuzzy-*I*-continuous, intuitionistic fuzzy semi-*I*-continuous.

2. Preliminaries

Definition.2.1.[3, 4, 5] LetX is a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form $A= \{<x,\mu_{A(x)},\nu_{A(x)}> : x\in X\}$ where the function $\mu_A : X\to [0, 1]$ and$\nu_A : X\to [0, 1]$ denote the degree of membership (namely $\mu_{A(x)}$) and the degree of non-membership (namely $\nu_{A(x)}$) of each element $x\in X$ to the set A, respectively and $0\leq \mu_{A(x)}+ \nu_{A(x)} \leq 1$ for each $x\in X$.

Remark.2.1. For the sake of simplicity, me shall use the symbol $A= <x, \mu_A, \nu_A>$ for the IFS $A= \{<x,\mu_{A(x)}, \nu_{A(x)}> : x\in X\}$.

Definition.2.2.[3] $1_\sim= \{<x,1, 0> : x\in X\}$and $0_\sim= \{<x,0, 1> : x\in X\}$.

Definition.2.3.[3]Let A and B are IFSs of the form $A= \{<x,\mu_{A(x)}, \nu_{A(x)}> : x\in X\}$ and

$B= \{<x,\mu_{B(x)}, \nu_{B(x)}> : x\in X\}$. Then;

i) $A\subseteq B$ if and only if$\mu_{A(x)} \leq\mu_{B(x)}$ and $\nu_{A(x)}\geq\nu_{B(x)}$ for every $x\in X$

ii) $\overline{A}= \{<x,\nu_{A(x)}, \mu_{A(x)}>: x\in X\}$

iii) $A\cap B = \{<x,\mu_{A(x)} \wedge \mu_{B(x)}, \nu_{A(x)} \vee \nu_{B(x)}>: x\in X\}$

iv) $A\cup B= \{<x,\mu_{A(x)} \vee \mu_{B(x)}, \nu_{A(x)} \wedge \nu_{B(x)}>: x\in X\}$

Definition.2.4.[6] An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

$T_1)$ $1_\sim ,0_\sim \in\tau$,

$T_2)$ $G_1\cap G_2\in\tau$ for any $G_1, G_2\in\tau$,

$T_3)$ $\cup G_i\in\tau$ for any arbitrary family $\{G_i :i\in\tau \}\subseteq\tau$.

In this case the pair (X, τ) is called an intuitionistic

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fuzzy topological space (IFTS for short) and any IFS in τ is known as anintuitionistic fuzzy open set (IFOS for short) in X.

Definition.2.5.[7]The complement C(A) of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short in X).

Definition.2.6.[7] Let (X, τ) IFTS and A= <x, μ_A, ν_A> be IFS in X. Then the intuitionisticfuzzy interior and intuitionisticfuzzy closure of A are defined by :

Int(A)=∪{G: G is an IFOS in X and G ⊆ A},

Cl(A)=∩{K: K is an IFCS in X and A ⊆ K}.

Deifintion.2.7.[8] A nonempty collection of intuitionistic fuzzy sets of a set X is called intuitionistic fuzzy ideal on X if;

i) A∈*I*and B ⊆ A ⇒B∈*I*(heredity),

ii) A∈*I*and B∈*I*⇒A∪B∈*I*(finite additivity).

Defintion.2.8.[8] Let (X, τ) be an intuitionistic fuzzy topological space (IFTS for short) and let A be in intuitionistic fuzzy (IF*I* for short) on X. Let A be any IFS of X. then the intuitionistic fuzzy local function A*(*I*, τ) of A is the union of all intuitionistic fuzzy points (IFP for short) C(α, β) such that if U∈ N(C(α, β)) and
A*(*I*, τ) = ∪{C(α, β) ∈ X: A∩U ∉ *I*for every U∈ N(C(α, β))}.
A*(*I*, τ) is called an intuitionistic fuzzy local function of A with respect to τ and *I* which it will be denoted by A*(*I*, τ), or simply A*(*I*).

Theorem.2.1.[8] Let (X, τ) be an IFTS and I_1, I_2 be two intuitionistic fuzzy ideals on X. Then for any intuitionistic fuzzy sets A, B of X. Then the following statement are verified

i) A ⊆ B⇒A*(*I*, τ) ⊆ B*(*I*, τ),
ii) I_1 ⊆ I_2⇒A*(I_1, τ) ⊆ B*(I_2, τ),
iii) A* =Cl(A*) ⊆ Cl(A),
iv)A^{**} ⊆ A* ,
v)(A∪B)*= A*∪B*,
vi) (A∩B)* ⊆ A*∩B*,
vii) l∈*I*⇒(A∪l)*= A*
viii) A*(*I*, τ) is fuzzy closed set.

Theorem.2.2.[8] Let τ_1, τ_2 be two intuitionistic fuzzy topologies on X. Then for any intuitionistic fuzzy ideal *I* on X, $\tau_1 \subseteq \tau_2$ implies

i) A*(*I*, τ_1) ⊆ A*(*I*, τ_2), for every A∈*I*,
ii) τ_1* ⊆ τ_2*.

Theorem.2.3.[8] β(*I*,τ)={A-B : A∈τ, B∈*I*} forms a basis for the generated IFT-τ* of the IFT(X, τ) with intuitionistic fuzzy ideal *I* on X.

Defintion.2.9.[8]Let (X, τ) be an IFTS and *I* be IF*I* on X. Let us defineintuitionistic fuzzy closure operator Cl*(A)= A∪A* for any IFS A of X.

Defintion.2.10.[8]Let (X, τ) be an IFTS and *I* be IF*I* on X. The IFTS with any *I*be IFI is said intuitionistic fuzzy ideal topological space(X, τ,*I*).

3. Intuitionistic fuzzy-*I*-open sets

Definition.3.1. A subset A of an intuitionistic fuzzy ideal topological space (X, τ, *I*) is said to be intuitionistic fuzzy-*I*-open set ifA ⊆Int(A*).We denote the family of all intuitionistic fuzzy-*I*- open sets of an intuitionistic fuzzy ideal topological space (X, τ, *I*) by IF*I*O(X). Also, the intuitionistic fuzzy ideal interior of A, denoted by IF*I*-int(A), is defined by the fuzzy union of all intuitionistic fuzzy-*I*-open set contained in A.

Example.3.1. Let X = {a, b, c} and

A=<x, $(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.6})$, $(\frac{a}{0.4},\frac{b}{0.3},\frac{c}{0.2})$>

B=<x, $(\frac{a}{0.6},\frac{b}{0.1},\frac{c}{0.5})$, $(\frac{a}{0.4},\frac{b}{0.7},\frac{c}{0.3})$>

We put τ={$0_\sim$, $1_\sim$,A}. If we take *I*={$0_\sim$}, then B∈IF*I*O(X).

Theorem.3.1. In (X, τ, *I*), A∈ IF*I*O(X) iff there exists U∈τ such that A⊆ U⊆ A*(*I*).

Proof. Necessity: Let A∈IF*I*O(X). Then A⊆Int(A*) = U ⊆ A*.

Sufficiency: Let A⊆U⊆ A*. Then A⊆ U=Int(U)⊆Int(A*). Hence, A∈IF*I*O(X).

Any subclass μ of I^X is called intuitionistic fuzzy supra topology [7] on X if μ contains $0_\sim$, $1_\sim$and closed under arbitrary union.

Theorem.3.2. For a space (X, τ, *I*), the class of IF*I*O(X) forms an intuitionistic fuzzy supra topology.

Proof.i) It is obvious that $0_\sim$ and X is IF*I*O(X).
ii) Let {A_α : α∈∇} be a class of IF*I*O(X). Then for any α∈∇, A_α⊆Int(A_α*). So
$\cup_{\alpha\in\nabla} A_\alpha \subseteq \cup_{\alpha\in\nabla}(\mathrm{Int}(A_\alpha^*)) \subseteq \mathrm{Int}(\cup_{\alpha\in\nabla} A_\alpha^*)$
$=\mathrm{Int}(\cup_{\alpha\in\nabla} A_\alpha)^*$. This completes the proof.

Theorem.3.3.[8]For any (X, τ, *I*) if U ⊆ X and V ∈τ then

Fadhil Abbas

i) $Cl(U^*(I)) \subseteq Cl(U)$,
ii) $V \cap U^*(I) \subseteq (V \cap U)^*(I)$.

Theorem.3.4. The intersection of any intuitionistic fuzzy open and intuitionistic fuzzy-*I*-open is intuitionistic fuzzy-*I*-open.

Proof. Since $U \in \tau$ and $V \in IFIO(X)$ in (X, τ, I), then $U \cap V \subseteq U \cap Int(U \cap V^*) \subseteq Int(U \cap V)^*$.

Proposition.3.1. Let (X, τ, I) be a space with $A \in X$. Then $IFI\text{-}Int(A) = A \cap Int(A^*)$.

Proof. Since $Int(A^*) = A^* \cap Int(A^*) \subseteq (A \cap Int(A^*))^*$, this leads to $A \cap Int(A^*) \subseteq Int(A \cap Int(A^*))^*$. Then $A \cap Int(A^*) \in IFIO(X)$, i.e., $A \cap Int(A^*) \subseteq IFI\text{-}Int(A)$. For the reverse inclusion if $U \in IFIO(X)$ and $U \subseteq A$, then Theorem.3.5. of [9] shows that $U^*(I) \subseteq A^*(I)$ and so, $Int(U^*) \subseteq Int(A^*)$ which gives $U = U \cap Int(U^*) \subseteq A \wedge Int(U^*)$. Hence the result.

Definition.3.2. For a space (X, τ, I) if $A \subseteq X$, the intuitionistic fuzzy restriction of I to A, which is denoted by $I_{/A}$ and defined as:
$I_{/A} = \{E \cap W : E \in I\}$. $I_{/A}$ is intuitionistic fuzzy ideal. By the intuitionistic fuzzy restriction of I on A, the intuitionistic fuzzy relative topology of τ on A, denotedby $\tau_{/A} = \{A \cap U : U \in \tau\}$.

One of the important properties concerning with the fuzzy local function with the fuzzy restriction ideal can be established through the following result which is having a straightforward proof.

Proposition.3.2. Let (X, τ, I) be space with $A \subseteq B \subseteq X$. Then $A^*(I_{/B}, \tau_{/B}) = A^*(I, \tau) \cap B$.

Theorem.3.5. In (X, τ, I) if $A \in IFIO(X)$ and $U \subseteq A$, then $Int_A(U^*(I_{/A}, \tau_{/A}) = A \cap Int(A^*(I, \tau))$, where Int_A denotes the intuitionistic fuzzy interior operator with respect to $\tau_{/A}$.

Proof. Let $p \in A \cap Int(U^*(I))$. Then there exists $V \in \tau(P)$ such that $p \in A \cap V \subseteq A \cap U^*(I)$
$= U^*(I_{/A}, \tau_{/A})$. Hence $p \in Int_A(U^*(I_{/A}, \tau_{/A}))$. For the reverse inclusion, let $p \in Int_A(U^*(I_{/A}, \tau_{/A}))$. Then there exists $V \in \tau(P)$ such that $p \in V \cap A \subseteq U^*(I_{/A}, \tau_{/A})$
$= A \cap U^*(I, \tau)$. Now, we have
$(V \cap A)^*(I) \subseteq (A \cap U^*(I, \tau))^*(I) \subseteq A^*(I) \cap (U^*(I))(I) \subseteq A^*(I) \cap U^*(I)$. Since $U \subseteq A$,
$A^*(I) \cap (U^*(I) = U^*(I)$.
Thus, $Int(V \cap A)^*(I) \subseteq Int(U^*(I))$.
By Theorem.3.4. $V \cap A = IFIO(X)$ So
$V \cap A \subseteq Int(V \cap A)^*(I) \subseteq Int(U^*(I))$, which means that $p \in A \cap Int(U^*(I))$. Hence, the equality is satisfied.

Definition.3.3. A subset F of a space (X, τ, I) is said to be intuitionistic fuzzy-*I*-closed if its complement is intuitionistic fuzzy-*I*-open. The collection of all intuitionistic fuzzy-*I*-closed sets in (X, τ, I) will be denoted by $IFIC(X)$.

Remark.3.1. For subset A of (X, τ, I), we have $1-(Int(A^*)) \neq Int((1-A)^*)$ as shown by the following example.

Example.3.2. In Example.3.1, if we take $I = P(X)$ then B satisfies the above properties.

4. Intuitionistic fuzzy semi-I-open sets

Definition.4.1. A subset A of an intuitionistic fuzzy ideal topological space (X, τ, I) is said to be intuitionistic fuzzy semi-*I*-open set if $A \subseteq Cl^*(Int(A))$. We denote the family of all intuitionistic fuzzy semi-*I*-open sets of an intuitionistic fuzzy ideal topological space (X, τ, I) by $IFSIO(X)$.

Theorem.4.1. In (X, τ, I) the following statement holds:

i) Every intuitionistic fuzzy open set is intuitionistic fuzzy semi-*I*-open,
ii) Every intuitionistic fuzzy semi-*I*-open set is intuitionistic fuzzy semi-open.

Proof. i) Let A be an intuitionistic fuzzy open set. Then we have $A = Int(A) \subseteq Int(A) \cup (Int(A))^*$

$= CI^*(Int(A))$. Therefore, $A \in IFSIO(X)$.

ii) Let $A \in IFSIO(X)$. Then (Theorem.2.1.(iii)), $A \subseteq CI^*(Int(A)) = Int(A) \cup (Int(A))^*$
$\subseteq Int(A) \cup CI(Int(A)) = CI(Int(A))$.
Therefore, $A \in IFSO(X)$.

Remark.4.1. The converse of Theorem.4.1.is false in general as shown by the following example.

Example.4.1. Let $X = \{a, b, c\}$ and

$A = \langle x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.2}) \rangle$

$B = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.2}) \rangle$

We put $\tau = \{0_\sim, 1_\sim, A\}$. If we take $I = \{0_\sim\}$, then $B \in IFSIO(X)$, but B is not intuitionistic fuzzy open, since $B^* = Cl(B)$. If we take $I = P(X)$, then $B \in IFSO(X)$, but $B \notin IFSIO(X)$.

Remark.4.2. Intuitionistic fuzzy ideal open set and intuitionistic fuzzy semi-*I*-open set are independent notions. In Example.4.1., $A \in IFSIO(X)$, but A is not intuitionistic fuzzy-*I*-open with $I = P(X)$.

Example.4.2. Let $X = \{a, b, c\}$ and

A=<x, $(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.7})$, $(\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.1})$>

B =<x, $(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.4})$, $(\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.2})$>

C =<x, $(\frac{a}{0.1}, \frac{b}{0.9}, \frac{c}{0.8})$, $(\frac{a}{0.7}, \frac{b}{0.1}, \frac{c}{0.1})$>

We put τ={$0_\sim$, $1_\sim$,A, B, A∪B, A∩B}. If we take *I*={$0_\sim$}. Then C∈IF*I*O(X), but C∉ IFS*I*O(X). Because, Int(C*) = $1_\sim$, but Int(C) = $0_\sim$.

Theorem.4.2. In (X, τ, *I*), A is intuitionistic fuzzy semi-*I*-open if and only if there exists U ∈τ such that
U ⊆ A ⊆ Cl*(U).

Proof. Let A∈IFS*I*O(X). Then we have A⊆CI*(Int(A)). Take Int(A) = U. Then U ⊆ A ⊆ CI*(U). Conversely, let U be an intuitionistic fuzzy open set such that U⊆A ⊆ CI*(U). Since U⊆ A, U ⊆Int(A) and hence CI*(U) ⊆ CI*(Int(A)). Thus, we obtainA ⊆ CI*(Int(A)).

Theorem.4.3.A subset A of space (X, τ, *I*) is intuitionistic fuzzy semi-*I*-open if and only if CI*(A) = CI*(Int(A)).

Proof. Let A∈ IFS*I*O(X). We have A ⊆CI*(Int(A)). Then CI*(A) ⊆ CI*(Int(A)) and hence
CI*(A) = CI*(Int(A)). The converse is obvious.

Theorem.4.4. If A is an intuitionistic fuzzy semi-*I*-open set in a space (X, τ, *I*) and
A ⊆ B ⊆ CI*(A), then B is intuitionistic fuzzy semi-*I*-open in (X, τ, *I*).

Proof. Since A∈IFS*I*O(X), there exists an open set U such that U ⊆ A ⊆CI*(U).
By Theorem 4.2, we obtainB ∈IFS*I*O(X).

Theorem.4.5. For a space (X, τ, *I*), the class of IFS*I*O(X) forms an intuitionistic fuzzy supra topology.

Proof.i) It is obvious that $0_\sim$ and X is IFS*I*O(X).
ii) Let {A_α : α∈ ∇ } be a class of IFS*I*O(X). Then for any α∈ ∇ , A_α⊆ CI*(Int(A_α)).
Thus by using Lemma 3.1 in [7],$\cup_{\alpha\in\nabla}A_\alpha \subseteq \cup_{\alpha\in\nabla}$CI*(Int($A_\alpha$))=$\cup_{\alpha\in\nabla}$((Int($A_\alpha$))*∪(Int ($A_\alpha$))⊆$\cup_{\alpha\in\nabla}$(Int($\cup_{\alpha\in\nabla}A_\alpha$))*∪ (Int($\cup_{\alpha\in\nabla}A_\alpha$))= Cl*(Int($\cup_{\alpha\in\nabla}A_\alpha$)). This completes the proof.

Theorem.4.6. The intersection of any intuitionistic fuzzy open set and intuitionistic fuzzy semi-*I*-open set is intuitionistic fuzzy semi-*I*-open.
Proof. Since U∈τ and V ∈IFS*I*O(X) in (X, τ, *I*), thenU∩V⊆U∩CI*(Int(V))=U∩(Int(V) ∪(Int(V))*) = (U∩Int(V))∪(U∩(Int(V))*) = Int(U∩V)∪(U∩Int(V))*
= Int(U∩V)∪(Int(U ∩V))* = CI*(Int(U∩V)). Therefore, U∩V ∈IFS*I*O(X).

Proposition.4.1. Let (X, τ, *I*) be an intuitionistic fuzzy ideal topological space. If U∈τandW ∈IFS*I*O(X); then U∩W ∈IFS*I*O(U, $I_{/U}$, $\tau_{/U}$).

Lemma.4.1.[7]. Let X and Y be of intuitionistic fuzzy topological space such that X is product related to Y. Then for intuitionistic fuzzy sets A of Xand B of Y,
i) CI(A× B) = CI(A) × CI(B),
ii) Int(A× B) = Int(A) × Int(B).

Theorem.4.7. Let X and Y be of (X, τ, *I*)such that X is product related to Y. Then the product A× B of an intuitionistic fuzzy semi-*I*-openset A and intuitionistic fuzzy semi-*I*-open set B of Y is an intuitionistic fuzzy semi-open of intuitionistic fuzzy product space X× Y.
Proof. Since A and B are intuitionistic fuzzy semi-*I*-open sets, we have A ⊆ CI*(Int(A)) and B⊆ CI*(Int(B)). From Lemma.4.1.,we obtain A× B ⊆ CI*(Int(A)) × CI*(Int(B)) ⊆ CI(Int(A))× CI(Int(B)) = CI(Int(A)× Int(B)) =CI(Int(A× B)).Therefore, A× B is intuitionistic fuzzy semi-open.

Theorem.4.8. Let (X, τ, *I*)be anintuitionistic fuzzy ideal topological space and A ⊆ X. If*I* = {$0_\sim$}, then intuitionisticfuzzy semi-*I*-open set and intuitionisticfuzzysemi-open set are equivalent.
Proof. It is obvious, since A* = Cl(A).

Definition.4.2.A subset F of a space (X, τ, *I*) is said to be intuitionistic fuzzy semi-*I*-closed if its complement is intuitionistic fuzzy semi-*I*-open.The collection of all intuitionistic fuzzy semi-*I*-closed sets in (X, τ, *I*) will be denoted by IFS*I*C(X).

Remark.4.3. For subset A of (X, τ, *I*), we have 1 - Int(CI*(A)) ≠CI*(Int(1 - A)) as shown by the following example.

Example.4.2. InExample.4.1, if we take *I* = P(X) then A satisfies the above properties.

Theorem.4.9. If a subset A of (X, τ, *I*) is intuitionistic fuzzy semi-*I*-closed, then
Int(CI*(A)) ⊆ A.
Proof. Since A∈IFS*I*C(X), 1-A ∈IFS*I*O(X). Hence, 1-A ⊆ CI*(Int(1-A)) ⊆CI(Int(1-A))
= 1-Int(CI(A)) ⊆ 1-Int(CI*(A)). Therefore, we obtain Int(CI*(A)) ⊆ A.

Corollary.4.1. A be a subset of (X, τ, *I*)such that 1-Int(CI*(A)) = CI*(Int(1- A)). Then A is intuitionistic fuzzy semi-*I*-closed if and only if Int(CI*(A)) ⊆A.

5.Decomposition of intuitionistic fuzzy-I-continuity

Definition.5.1. An intuitionistic fuzzy set A of (X, τ, *I*)X is called a S-set if
CI*(Int(A)) = Int(A).

Definition.5.2. An intuitionistic fuzzy set A of (X, τ, *I*)X is called a S-set if A = U∩V, whereU ∈τ and V is a S-set of X.

Theorem.5.1. A intuitionistic fuzzy set A of (X, τ, *I*)is open if and only if it is both an intuitionistic fuzzy semi-*I*-open and a S-set.

Proof. Let A is an intuitionistic fuzzy open of X. Then A = A∩X follows that A is an intuitionistic fuzzy S-set. Also A is an intuitionistic fuzzy semi-*I*-open by Theorem.4.1(i).Conversely, Let A be both B intuitionistic fuzzy S-set and an intuitionistic fuzzy semi-*I*-open set. Then A ⊆ CI*(Int(A)) and A = U∩V, whereU ∈τ and V is a S-set of X. Therefore, U∩V ⊆ CI*(Int(U∩V)) ⊆ CI*(Int(U))∩CI*(Int(V)) = CI*(Int(U))∩Int(V)).Hence, U∩V = (U∩V)∩U ⊆CI*(Int(U))∩Int(V)∩U = U∩Int(V). Therefore, we obtain U∩V = U∩Int(V), thus A = U∩V is anintuitionistic fuzzy open set.

Definition.5.3. A mapping f :(X, τ, *I*)→ (Y,σ) is called intuitionisticfuzzy-*I*-continuous if
f^{-1}(V) is an intuitionistic fuzzy-*I*-open for each V ∈ σ.

Definition.5.4. A mapping f :(X, τ, *I*)→ (Y,σ) is called intuitionistic fuzzy semi-*I*-continuous (intuitionistic fuzzy S-continuous) if f^{-1}(V) is an intuitionistic fuzzy semi-*I*-open set (intuitionistic fuzzy S-set) for each V ∈ σ.

According to Theorem.5.1, we have the following decomposition of intuitionistic fuzzy continuity.

Theorem.5.2. A mapping f : (X, τ, *I*)→ (Y,σ) is intuitionistic fuzzy continuous if and only if it is both intuitionistic fuzzy semi-*I*-continuous and intuitionistic fuzzy S-continuous.

Theorem.5.3. Let f :(X, τ, *I*)→ (Y,σ, *J*) and g: (Y, σ, *J*)→ (Z,φ)be a mappings X, Y and Z are in intuitionistic fuzzy ideal topologicalspace. If f is semi-*I*-continuous and g is intuitionistic fuzzy continuous, then gof is intuitionistic fuzzy semi-*I*-continuous.

CONCLUSION

The purpose of this work is introduce a new class of intuitionistic fuzzy ideal open sets in intuitionistic fuzzy ideal topological spaces; intuitionistic fuzzy-*I*-open set and intuitionistic fuzzy semi-*I*-open set and given a decomposition of fuzzy continuity.

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